Assessing Secondary Teachers' Algebraic Habits of Mind

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TODAY'S AGENDA

1. History of our work
2. Paper and pencil assessment
   (a) Review items in small groups
   (b) Whole group discussion
3. Classroom observations
4. Wrap-up

WHAT IS ASTAHM?

ASTAHM is an NSF DRK-12 collaborative project funded in 2012 aimed at developing instruments to assess secondary teachers' Mathematical Habits of Mind (MHoM).

What do we mean by Mathematical Habits of Mind?

We define mathematical habits of mind (MHoM) to be:

> *the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways employed by mathematicians.*

KNOWING MATHEMATICS AS A MATHEMATICIAN

From our experience, we believe that knowing mathematics as a mathematician. . .

- enriches and enhances the other ways of knowing mathematics,
- can bring efficiency and coherence to teachers' mathematical thinking and to their work with students,
- and thus is an important aspect of mathematical knowledge for teaching at the secondary level.

Focus on Mathematics

Focus on Mathematics (FoM) is a partnership originally funded by the NSF (2003-2013). The goals of FoM are to:

- Provide teachers with:
  - coherent, content-focused professional development,
  - sustained immersion in mathematics,
- Develop mathematically expert teachers who share their knowledge with teachers and students,
- Build a mathematical learning community in which teachers and mathematicians work together, doing mathematics, and
- Improve student achievement.

INITIAL MOTIVATION FOR RESEARCH

- Through our FoM work, we've seen that MHoM is indeed a collection of habits teachers can acquire, rather than some static you-have-it-or-you-don't way of thinking.
- And teachers reported to us that developing these habits has had a tremendous effect on their teaching.
- We recognized the need for scientific-based evidence to establish that these teachers have indeed learned MHoM and that these habits have had a positive impact on their teaching practice.
- The instruments to measure these habits did not yet exist.

RESEARCH QUESTION

We began an exploratory study centered on the following research question:

> *What are the mathematical habits of mind that secondary teachers use, how do they use them, and how can we measure them?*

INSTRUMENTS FOR CONDUCTING RESEARCH

To investigate our research question, we've been developing:

- Detailed definition of MHoM, based on existing literature, our own experiences as mathematicians, and classroom observations.
- A paper and pencil (P&P) assessment that measures how teachers engage MHoM when doing mathematics for themselves.
- An observation framework for understanding the nature of teachers' use of MHoM in their classroom work.

Important remark: We've seen the value of developing all three components together.

What we aren't studying

There are many aspects of teaching that we value but we are *not* studying right now. For example:

- Teachers' dispositions (at least not directly)
- Teachers' beliefs
- Classroom discourse

What we aren't creating

We are *not* creating assessment tools that we anticipate can say much about an individual teacher. **Our goal is to create tools for research.**

Focus on MHoM

Our current focus is on three categories of MHoM:

- Engaging with one's experiences (EXPR)
- Making use of structure to solve problems (STRC)
- Using mathematical language precisely (LANG)

Note: Eventually, we will investigate other habits.

P&P ASSESSMENT: KEY FEATURES

Distinguishing features of the P&P assessment:

- **It measures how secondary teachers use MHoM in their own doing of mathematics, in familiar contexts.**
- Content is from secondary mathematics—i.e., mathematical problems that most teachers have the requisite knowledge to solve, or at least begin to solve.
- **We're interested in their approach, as opposed to whether or not they can arrive at a solution.**
- Our items are drawn from multiple sources, including our classroom observation work.

Maximum Value

Sample Item:

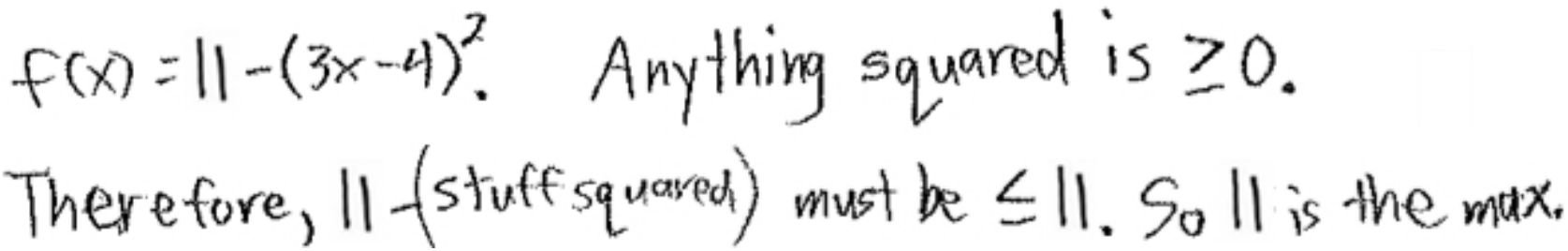
Find the maximum value of the function $f(x) = 11 - (3x - 4)^2$.

- Though most teachers obtained the same (correct) answer, there were vast variations in their approaches.
- These various approaches came in "clumps," as our advisors (assessment experts) and research literature had told us to expect.
- Using these responses, we developed a rubric that allows us to code how each teacher solved the problem.

SAMPLE CODE: SQUR

(SQUR) Since $(3x-4)^2$ represents the *square* of some number, it is always ≥ 0. Thus in the function $f(x) = 11 - (3x-4)^2$, we are always subtracting a non-negative number from 11. To maximize $f(x)$, we need $(3x-4)^2 = 0$ so the max value is 11.

Sample solution:



QUICK MATHEMATICAL NOTE

The reasoning described in SQUR depends on the fact that x can be chosen so that $(3x-4)^2 = 0$. In many cases, we had no way of knowing whether the teachers actually noticed this detail.

SAMPLE CODE: SYMM

(SYMM) Expanded $f(x)$ into $f(x) = -9x^2 + 24x - 5$. Found the axis of symmetry using the formula $x = -b/(2a) = 4/3$. Evaluated $f(4/3) = 11$ to obtain the maximum value.

Sample solution:

$f(x) = 11 - (3x-4)^2$

$= 11 - (9x^2 - 24x + 16)$

$= 11 - 9x^2 + 24x - 16$

$= -9x^2 + 24x - 5$

$f\left(\frac{4}{3}\right) = 11 - \left(3\left(\frac{4}{3}\right) - 4\right)^2$

$= 11 - (4-4)^2$

$= \boxed{11}$

x-coord. of vertex:

$\frac{-b}{2a} = \frac{-24}{2(-9)} = \frac{-24}{-18} = \frac{4}{3}$

max value is 11.

FOR YOU TO DO

In small groups of ~4 people, please review the P&P items assigned to your group and discuss the guiding questions in the handout.

- Group A: HYB
- Group B: PAT
- Group C: THP

Questions for the small groups:

- Is it clear what the question is asking? How could we make it more clear?
- Is the item accessible to secondary teachers? Is it relevant?
- Is the item interesting enough to engage you in mathematical thinking?
- What MHoM does the item capture?

Afterwards, we'll ask groups to report on your conversations.

HYB

To subtract a larger number from a smaller number, such as $38 - 72$, we typically "switch and negate." We first compute $72 - 38 = 34$, then negate this difference, so that $38 - 72 = -34$ (which is correct). Here is another approach, using the standard subtraction algorithm:

$$\begin{array}{r} 38 \\ -\,72 \\ \hline -46 \end{array}$$

Here, we first look at the ones place and compute $8 - 2 = 6$. Then we look at the tens place and find $3 - 7 = -4$. Lining them up, we obtain -46 (which is incorrect). Explain the mathematical error in this approach, i.e., why does it result in an incorrect answer?

Note: Hy Bass suggested a version of this item.

PAT

Suppose you have a sequence

$$a(0),\ a(1),\ a(2),\ a(3),\ a(4),\ \ldots$$

with $a(0) = m$ and $a(1) = n$. Each subsequent term in the sequence is the difference of the two previous terms. For example, $a(2) = a(1) - a(0)$, and $a(3) = a(2) - a(1)$, and so on. Find $a(1000)$ in terms of m and n. Explain your thinking.

Note: A version of this item first appeared in a classroom observation.

THP

Use the fact that $1764 \times 1765 = 3113460$ to find 1762×1767. Explain your thinking.

Note: This item was inspired by field tests of a new EDC curriculum, *Transition to Algebra*.

OBSERVING FOR MHOM

- Our observation goal is: evidence of *how* teachers' use of MHoM in their instruction.
- We are not looking for possession of certain habits in the abstract, but for how teachers choose to bring them to bear in a classroom.
- The framework targets the same mathematical habits as the P&P assessment.

GOALS OF THE OBSERVATION WORK

- To collect examples of how teachers use MHoM in the classroom, with an eye towards describing more generally what we mean by classroom use of MHoM.
- To study the bridge factors between teachers' own MHoM (as measured by the P&P assessment) and their use of these habits in the classroom.

For you to do

We will show a short video clip from an Algebra 1 classroom.

- What evidence of MHoM do you see in action?
- What evidence do you see of the teacher offering students opportunities to develop MHoM?

Fitting into the Practice Standards

We see our habits as closely related to at four of the Common Core Standards for Mathematical Practice:

- MP2: Reason abstractly and quantitatively.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

LEARN MORE OR PARTICIPATE

Want to learn more?

- Come to our (small) workshop in Boston this summer.
- Participate in the research!

mhomresearch.edc.org

THANK YOU

- Thank you for your participation and feedback!
- If you have further feedback and/or questions, email us at:
  - Ryota Matsuura (matsuura@stolaf.edu)
  - Sarah Sword (ssword@edc.org)