Assessing Secondary Teachers' Algebraic Habits of Mind

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Today's agenda

1. Background on our work
2. Paper and pencil assessment
3. Observation protocol
4. Further discussion and questions

What is ASTAHM?

ASTAHM is an NSF DRK-12 collaborative project funded in 2012 aimed at developing instruments to assess secondary teachers' Mathematical Habits of Mind (MHoM).

WHAT WE MEAN BY MATHEMATICAL HABITS OF MIND

We define mathematical habits of mind (MHoM) to be:

> *the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways employed by mathematicians.*

MET2 FRAMEWORK

The MET2 framework uses four large and overlapping categories to characterize some of the ways in which teachers know and understand mathematics:

(1) As a scholar

(2) As an educator

(3) As a mathematician

(4) As a teacher

And we are studying the intersection of (3) and (4).

Focus on Mathematics

- Focus on Mathematics (FoM) is a Targeted Math and Science Partnership funded by the NSF since 2003.
- FoM is a unique partnership of teachers, students, administrators, mathematicians, and mathematics educators—all committed to increasing student achievement.
- FoM is a collaboration between BU, EDC, and an array of Massachusetts universities and public school districts.

FoM goals

- Provide teachers with:
  - coherent, content-focused professional development,
  - sustained immersion in mathematics,
- Develop mathematically expert teacher leaders who share their knowledge with teachers and students,
- Build a mathematical learning community in which teachers and mathematicians work together, doing mathematics, and
- Improve student achievement.

AUTHENTIC MATHEMATICAL EXPERIENCES

Experience first:

It has been observed in every human activity experience comes first, and as this experience grows the need for communication motivates the development of language. Sadly enough, in our classroom practice we place language first and experience second. We worry about what we should say in order to help the student "understand." By this we mean to provide the effect of experience through the use of suitably chosen words. Not unexpectedly, the effect is at best a very pale image of the real thing.

— Arnold Ross

Impact on teachers

FoM teachers have reported[1]:

- Deeper knowledge of mathematics
- Changes in beliefs about the nature of mathematics and how students learn mathematics
- Renewed passion for mathematics
- Changes in instructional practice, for example
  - use of precise language
  - connecting mathematical ideas
  - mining student ideas and approaches

[1] Baldassari, C., Lee, S., & Torres, R. T. (2009). *The case of a high school mathematics teacher.* Retrieved from http://focusonmath.org/FOM/PERG

INITIAL MOTIVATION FOR RESEARCH

- Through our FoM work, we've seen that MHoM is indeed a collection of habits teachers can acquire, rather than some static you-have-it-or-you-don't way of thinking.
- And teachers reported to us that developing these habits has had a tremendous effect on their teaching.
- We recognized the need for scientific-based evidence to establish that these teachers have indeed learned MHoM and that these habits have had a positive impact on their teaching practice.
- The instruments to measure these habits did not yet exist.

RESEARCH QUESTION

Our work is centered on the following research question:

> *What are the mathematical habits of mind that secondary teachers use, how do they use them, and how can we measure them?*

INSTRUMENTS FOR CONDUCTING RESEARCH

To investigate our research question, we've been developing:

- Detailed definition of MHoM, based on existing literature, our own experiences as mathematicians, and classroom observations.
- A paper and pencil (P&P) assessment that measures how teachers engage MHoM when doing mathematics for themselves.
- An observation protocol measuring the nature and degree of teachers' use of MHoM in their classroom work.

Important remark: We've seen the need for both instruments, and also the value of developing all three components together.

What we aren't studying

There are many aspects of teaching that we value but we are *not* studying right now. For example:

- Teachers' dispositions (at least not directly)
- Teachers' beliefs
- Classroom discourse

WHAT WE AREN'T CREATING

We are *not* creating instruments that we anticipate can say much about an individual teacher. **Our goal is to create tools for research.**

Assessment development

Since 2010:

- Several rounds of design, small scale pilot tests (w/ FoM community), data analysis, and revision of the instrument.
- Inter-rater reliability testing for some items (κ up to 0.906) and concurrent rubrics refinement.
- Problem-solving think-aloud with teachers, educators, and mathematicians. (E.g., do their interpretations match our intended objectives for each item?)

Pilot test in 2011

We conducted an “external” pilot test in 2011.

- 43 teachers ninth grade Algebra 1 teachers.
- Studying implementation of a new NSF (MHoM) curriculum, *CME Project*.
- 7 items (topics in secondary mathematics—algebra and geometry), to be completed in one hour.
- Cronbach alpha testing data suggests the instrument is reliable about 68% of the time.

ASTAHM PILOT TESTS 2012

We conducted four additional pilot tests in 2012.

- ~70 teachers and ~15 “others”
- Geographic diversity
- Two part assessment:
  - 10 short items to be completed in one hour
  - 1 longer item to be completed in 30 minutes

MHoM: Structure and language

Our current focus is on two categories of MHoM:

- Seeking, Using, and Describing Mathematical Structure (SUDS)
  - SUDS1. Discovering structure that is not apparent at first.
  - SUDS2. Making use of structure to solve problems.
- Using Mathematical Language (LANG)
  - LANG1. Using language to acquire experience, clarity, and understanding.
  - LANG2. Exercising appropriate “mathematical hygiene” (e.g., using language precisely).

Note: Eventually, we will investigate other habits, e.g., “Performing purposeful experiments” and “Applying mathematical reasoning.”

P&P ASSESSMENT: KEY FEATURES

Distinguishing features of the P&P assessment:

- **It measures how secondary teachers use MHoM in their own doing of mathematics, in familiar contexts.**
- Content is from secondary mathematics—i.e., mathematical problems that most teachers have the requisite knowledge to solve, or at least begin to solve.
- **We're interested in their approach, as opposed to whether or not they can arrive at a solution.**
- Our items are drawn from multiple sources, including our classroom observation work.

Maximum Value

Sample Item:

Find the maximum value of the function $f(x) = 11 - (3x - 4)^2$.

- Though most teachers obtained the same (correct) answer, there were vast variations in their approaches.
- These various approaches came in "clumps," as our advisors (assessment experts) and research literature[2] told us to expect.
- Using these responses, we developed a rubric that allows us to code how each teacher solved the problem.

[2]Erickson; Glasser & Strauss; Guba; Maxwell

SAMPLE CODE: SQUR

(SQUR) Since $(3x-4)^2$ represents the *square* of some number, it is always ≥ 0. Thus in the function $f(x) = 11-(3x-4)^2$, we are always subtracting a non-negative number from 11. To maximize $f(x)$, we need $(3x-4)^2 = 0$ so the max value is 11.

Sample solution:

$f(x) = 11-(3x-4)^2$. Anything squared is ≥ 0.

Therefore, 11 − (stuff squared) must be ≤ 11. So 11 is the max.

Quick mathematical note

The reasoning described in SQUR depends on the fact that x can be chosen so that $(3x-4)^2 = 0$. In many cases, we had no way of knowing whether the teachers actually noticed this detail.

SAMPLE CODE: SYMM

(SYMM) Expanded $f(x)$ into $f(x) = -9x^2 + 24x - 5$. Found the axis of symmetry using the formula $x = -b/(2a) = 4/3$. Evaluated $f(4/3) = 11$ to obtain the maximum value.

Sample solution:

$$
\begin{aligned}
f(x) &= 11 - (3x-4)^2 \\
&= 11 - (9x^2 - 24x + 16) \\
&= 11 - 9x^2 + 24x - 16 \\
&= -9x^2 + 24x - 5
\end{aligned}
$$

$$
\begin{aligned}
f\left(\tfrac{4}{3}\right) &= 11 - \left(3\left(\tfrac{4}{3}\right) - 4\right)^2 \\
&= 11 - (4-4)^2 \\
&= \boxed{11}
\end{aligned}
$$

x-coord. of vertex:

$$\frac{-b}{2a} = \frac{-24}{2(-9)} = \frac{-24}{-18} = \frac{4}{3}$$

max value is 11.

REVIEWING SOME ITEMS: SMALL GROUP DISCUSSIONS

Questions to consider during the discussion:

- Is it clear what the items are asking? If not, how could we make them more clear?
- Is the mathematics content *relevant* to secondary teaching?
- Is the mathematics content *accessible* to secondary teachers?
- Do the items capture the MHoM we are trying to capture?
- What other information would you like to see in the rubrics?

NEXT STEPS ON THE P&P

What's next?

- Upcoming rounds of smaller field tests (we just can't stop!)
- Validity and reliability testing of the assessment
- Larger field test 2014–2015: 200 teachers
- Student data collection (PARCC assessments)

Observation protocol: Key features

- Our observation protocol focuses on measuring *how* teachers use their MHoM in their instruction.
- Teachers are coded not for possessing certain habits in the abstract, but for choosing to bring them to bear in a classroom.
- The protocol targets the same mathematical habits as the P&P assessment. Today, we'll look in particular at LANG1.

Observation Protocol: Review Process

We reviewed 33 existing observation protocols selected from collections:

- Examined in Compendium of research instruments for STEM education (Minner, Martinez, & Freeman, 2012)
- Highly cited in literature (Henry, et al., 2009)
- Examined in a practitioner's guide (Dirr, ND)
- Investigated by the Measures of Effective Teaching (MET) study (Bill and Melinda Gates Foundation 2012)

Observation Protocol: Design Process

- Rating types:
  - Global (see Weiss, et al., 2003)
  - Discrete (see North Cascades and Olympic Science Partnership, 2008)
- Observation focus:
  - Teaching (see Danielson, 2012)
  - Content (see Piburn & Sawada's Reformed Teaching Observation Protocol)
  - Curriculum (see Coffey & Feldman, 2009)

VIDEO CLIP

We will show a short video clip from an Algebra 1 classroom, where the students are learning about the Zero Product Property.

Recall that the **Zero Product Property** states:

If $ab = 0$ then $a = 0$ or $b = 0$.

Observation protocol: Guiding questions

- How did you see the teacher using LANG1?
- Did you see the teacher using SUDS? If so, how?

YOUR TURN

Please describe a time in your own mathematical experience (either as a teacher or as a student of mathematics) when you relied on the habit LANG1, which says…

LANG1: Using language to acquire experience, clarity, and understanding. This habit is about using language to "force one's brain into clarity."

Thank you

- If you have further feedback and/or questions, please email us:

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