Measuring Secondary Teachers' Use of Standards for Mathematical Practice

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TODAY'S AGENDA

1. Background on our work
2. Paper and pencil assessment
   (a) Review the items in small groups
   (b) Whole group discussion
3. Further discussion and questions

What is ASTAHM?

ASTAHM is an NSF DRK-12 collaborative project funded in 2012 aimed at developing instruments to assess secondary teachers' mathematical habits of mind (MHoM).

WHAT DO WE MEAN BY MHOM?

We define mathematical habits of mind (MHoM) to be:

> *the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways employed by mathematicians.*

Focus on MHoM

Our current focus is on three categories of MHoM:

- **EXPR.** Engaging with one's experiences
- **STRC.** Making use of structure to solve problems
- **LANG.** Using mathematical language precisely

Remark: Focusing on three habits has allowed us to create instruments that are not too burdensome to use. Eventually, we will investigate other habits, too.

CONNECTION TO CCSSM

Our three mathematical habits are closely related to the following Common Core Standards for Mathematical Practice:

- **MP2.** Reason abstractly and quantitatively
- **MP6.** Attend to precision
- **MP7.** Look for and make use of structure
- **MP8.** Look for and express regularity in repeated reasoning

INITIAL MOTIVATION FOR RESEARCH

- Through our professional development work, we've seen that MHoM is indeed a collection of habits teachers can acquire, rather than some static you-have-it-or-you-don't way of thinking.
- Teachers report that developing these habits has a tremendous effect on their teaching.
- We recognize the need for scientific-based evidence to establish that teachers' MHoM are not static and that these habits have a positive impact on their teaching practice.
- Instruments to measure these habits have not existed.

RESEARCH QUESTION

What are the mathematical habits of mind that secondary teachers use, how do they use them, and how can we measure them?

INSTRUMENTS FOR CONDUCTING RESEARCH

To investigate our research question, we've been developing:

- Detailed definition of MHoM, based on existing literature, our own experiences as mathematicians, and classroom observations.
- A paper and pencil (P&P) assessment that measures how teachers engage MHoM when doing mathematics for themselves.
- An observation protocol measuring the nature and degree of teachers' use of MHoM in their classroom work.

Important remark: We've seen the need for both instruments, and also the value of developing all three components together.

What we aren't studying

There are many aspects of teaching that we value but we are *not* studying right now. For example:

- Teachers' dispositions (at least not directly)
- Teachers' beliefs
- Classroom discourse

WHAT WE AREN'T CREATING

We are *not* creating an assessment that we anticipate can say much about an individual teacher. **Our goal is to create tools for research.**

P&P ASSESSMENT: OVERVIEW

- We are developing a P&P assessment that measures how teachers engage MHoM when doing mathematics for themselves.
- The assessment has been field-tested with over 500 teachers. Field-tests are ongoing.
- Initial validity and reliability testing yielded promising results. More testing is being planned.
- Again, this is a tool for research, *not* for teacher evaluation.

P&P ASSESSMENT: KEY FEATURES

- Assessment measures how secondary teachers use mathematical habits of mind when doing mathematics.
- Items are accessible: most secondary teachers can solve them, or at least begin to solve them.
- Coding focuses on the *approach*, not on "the correct solution."
- Assessment items are drawn from multiple sources, including our classroom observation work.

MAXIMUM VALUE

Sample Item:

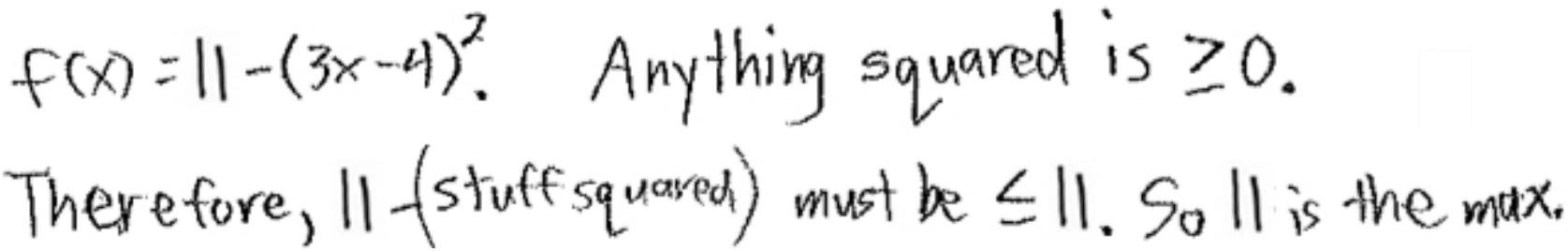
Find the maximum value of the function $f(x) = 11 - (3x - 4)^2$.

- Though most teachers obtained the same (correct) answer, there were vast variations in their approaches.
- These various approaches came in "clumps," as our advisors (assessment experts) and research literature had told us to expect.
- Using these responses, we developed a rubric that allows us to code how each teacher solved the problem.
- Sidenote: see CCSS.Math.Practice.MP7.

SAMPLE CODE: SQUR

(SQUR) Since $(3x-4)^2$ represents the *square* of some number, it is always ≥ 0. Thus in the function $f(x) = 11 - (3x-4)^2$, we are always subtracting a non-negative number from 11. To maximize $f(x)$, we need $(3x-4)^2 = 0$ so the max value is 11.

Sample solution:



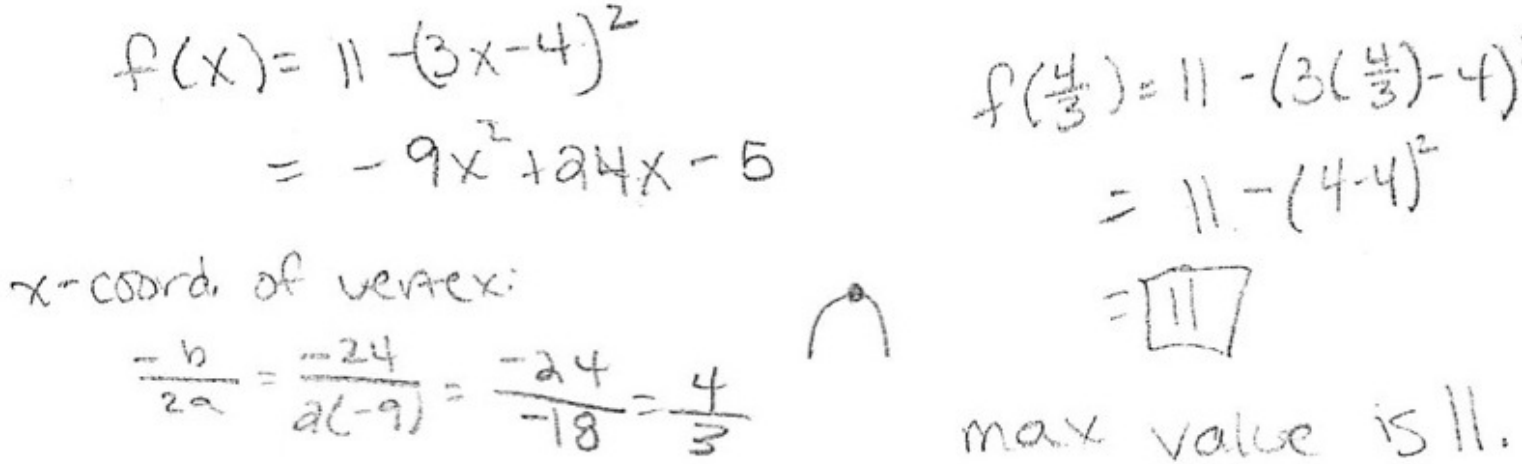
QUICK MATHEMATICAL NOTE

The reasoning described in SQUR depends on the fact that x can be chosen so that $(3x-4)^2 = 0$. In many cases, we had no way of knowing whether the teachers actually noticed this detail.

SAMPLE CODE: SYMM

(SYMM) Expanded $f(x)$ into $f(x) = -9x^2 + 24x - 5$. Found the axis of symmetry using the formula $x = -b/(2a) = 4/3$. Evaluated $f(4/3) = 11$ to obtain the maximum value.

Sample solution:



HYB[1]

To subtract a larger number from a smaller number, such as $38 - 72$, we typically "switch and negate." We first compute $72 - 38 = 34$, then negate this difference, so that $38 - 72 = -34$ (which is correct). Here is another approach, using the standard subtraction algorithm:

$$\begin{array}{r} 38 \\ -\,72 \\ \hline -46 \end{array}$$

Here, we first look at the ones place and compute $8 - 2 = 6$. Then we look at the tens place and find $3 - 7 = -4$. Lining them up, we obtain -46 (which is incorrect). Explain the mathematical error in this approach, i.e., why does it result in an incorrect answer?

[1] **Note:** Hy Bass suggested a version of this item.

DIG INTO THE ITEMS/RUBRICS

Please consider these questions as you review the items/rubrics:

- Where do you see MHoM being used in these approaches?
- Do the ways in which you think about this item match the habit that we claim it measures?
- How would you want students to approach this problem?
- What connections do you see to the four SMPs?
  - **MP2.** Reason abstractly and quantitatively
  - **MP6.** Attend to precision
  - **MP7.** Look for and make use of structure
  - **MP8.** Look for and express regularity in repeated reasoning

LEARN MORE OR PARTICIPATE

Want to learn more, use the assessment, or participate in the research?

mhomresearch.edc.org

THANK YOU

- Thank you for your participation and feedback!
- If you have further feedback and/or questions, email us at:
  - Sarah Sword (ssword@edc.org)
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