Secondary Teachers’ Mathematical Habits of Mind

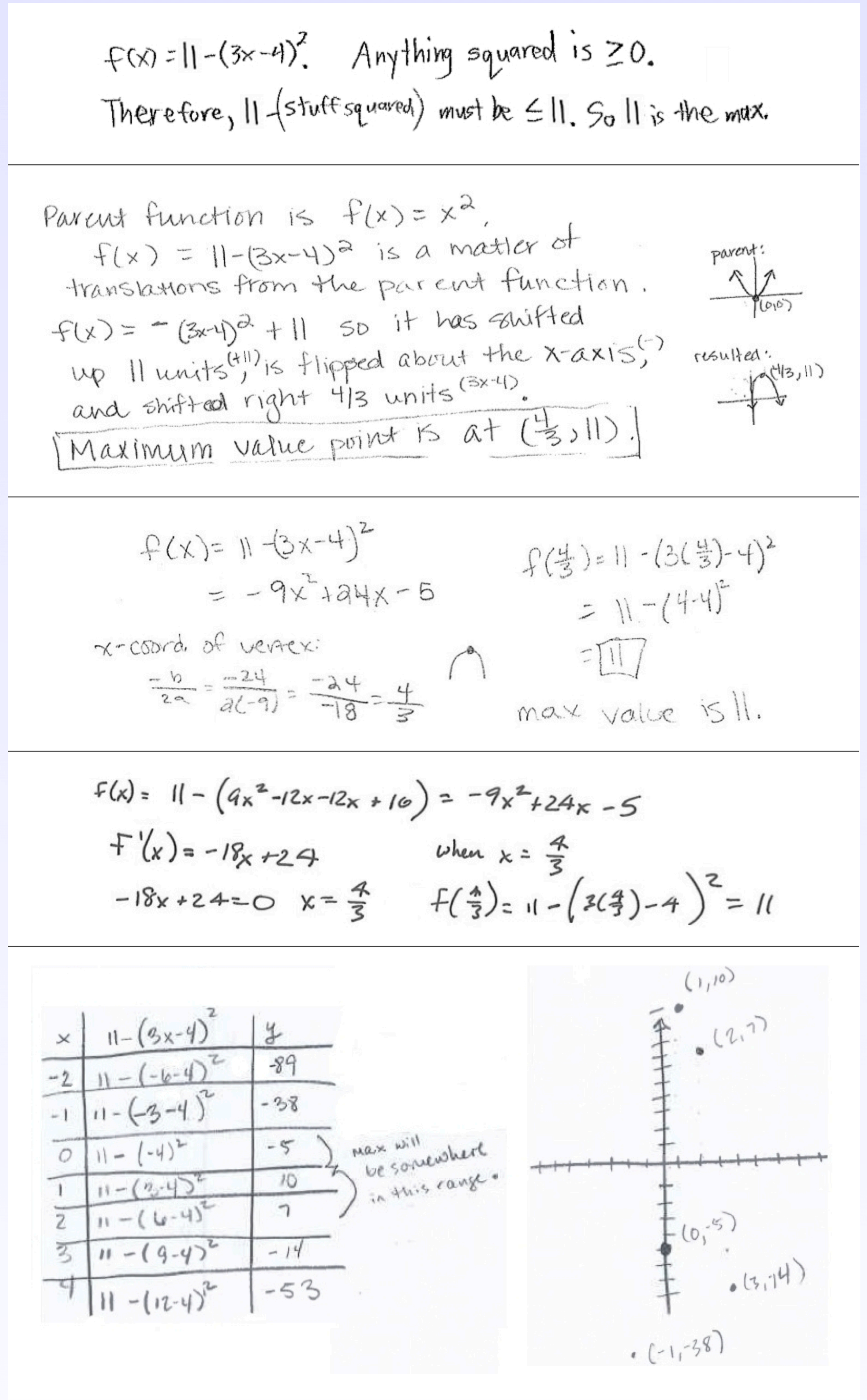
mhomresearch.edc.org

Paper and Pencil Assessment

- Assessment measures how secondary teachers use mathematical habits of mind when doing mathematics.
- Items are accessible: most secondary teachers can solve them, or at least begin to solve them.
- Coding focuses on the *approach*, not on “the correct solution.”
- Field tested with 500+ teachers across the country.
- Assessment designed for research, not for teacher evaluation.

Maximum Value item and sample approaches:

Find the maximum value of the function $f(x) = 11 – (3x – 4)^2$. Explain your reasoning.



Questions? Interested in using the assessment? Please contact Sarah Sword: ssword@edc.org

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What MHoM do secondary teachers use, how do they use them, and how can we measure them?

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How do teachers who engage MHoM when doing mathematics for themselves also bring MHoM to their teaching practice?

We define *mathematical habits of mind* to be the specialized ways of approaching mathematical problems and thinking about mathematical concepts that resemble the ways employed by mathematicians.

Targeting three mathematical habits of mind

Although many MHoM are worthy of study, focusing on three habits has allowed us to create an assessment that is not too burdensome to use.

Using mathematical structure. This habit entails taking advantage of the underlying structure of a given situation to facilitate problem solving.

For example, consider the task of simplifying the expression $3(99^2 – 1) + 8(99^2 – 1) – 11(99^2 – 1)$. By viewing $(99^2 – 1)$ as a common term, the expression becomes 3♧ + 8♧ – 11♧, which equals 0♧ or 0. This approach, which treats $(99^2 – 1)$ as a single object ♧, is called “chunking,” which entails intentionally avoiding certain details to hide the complexity of an algebraic expression.

Seeking mathematical structure. This habit is about the *search* for useful structures that are not immediately apparent. It typically involves experimental processes such as writing expressions in equivalent forms, working with smaller/special cases, seeking regularity in repeated calculations, and generalizing from concrete examples.

For example, consider the task: Use the fact that $1764 \times 1765 = 3113460$ to find 1762×1767. The original product has the form $x(x + 1) = x^2 + x$ and the new product has the form $(x – 2)(x + 3) = x^2 + x – 6$. Thus the latter is 6 less than the former, so that $1762 \times 1767 = 3113454$. This approach unravels the relationship between these products by representing them symbolically and comparing their forms. Another approach involves the use of smaller examples—e.g., start with 8×9 and 6×11 and notice that they differ by 6. This approach, however, does not *explain* the relationship between the products.

Using mathematical language clearly. This habit entails communicating clearly with others. Features of this habit may include (but are not limited to) understanding and appropriately using mathematical terminologies and definitions, using mathematical notation correctly, and giving clear descriptions of the steps in a process.

For example, a student asks about the question: Determine if $r = –2$ is a solution to $6r + 2 = 12 + r$. The teacher asks the class what “solution” means, to which students respond with phrases such as “when it works” and “the answer.” This imprecise language does not help unravel the problem to understand what they are being asked to do. Thus, the teacher encourages them to be more specific. Eventually, they arrive at, “something that makes the equation true.” The class can now *use* this definition to substitute $r = –2$ into the equation and check if it makes it true.

Original Field Study Results (n=274)

Statistic	Value	Interpretation
Chi-square	29.475 (p = 0.595)	Good. Indicates that the model fits the data well.
Root mean square error of approximation (RMSEA)	0.01	Excellent
Confirmatory fit index (CFI)	1.00	Excellent
GFI (Goodness of fit index)	0.98	Excellent
Tucker-Lewis index	1.01	Excellent

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Connections to Practice and Student Outcomes

Growth in MHoM in the context of mathematics and equity oriented PD was linked higher instructional quality as measured by the IQA (Boston, 2012).

		AR 3	AR X	AT 3	AT 4	AT5
Change in MHoM Summative	Pearson Correlation	.61*	.63*	.56*	.68**	.65*
	N	13	13	13	13	13
Change in MHoM LANG subscale	Pearson Correlation					.71**
	N					13

AR3: Academic Rigor; **ARX:** Mathematical Residue**;** **AT3:** Accountable Talk; **AT3:** Student linking; **AT4:** Teacher press; **AT5:** Student Responses.

Growth was also linked to student outcomes on the Tripod survey (Tripod Education Partners), a measure of students’ classroom experiences.

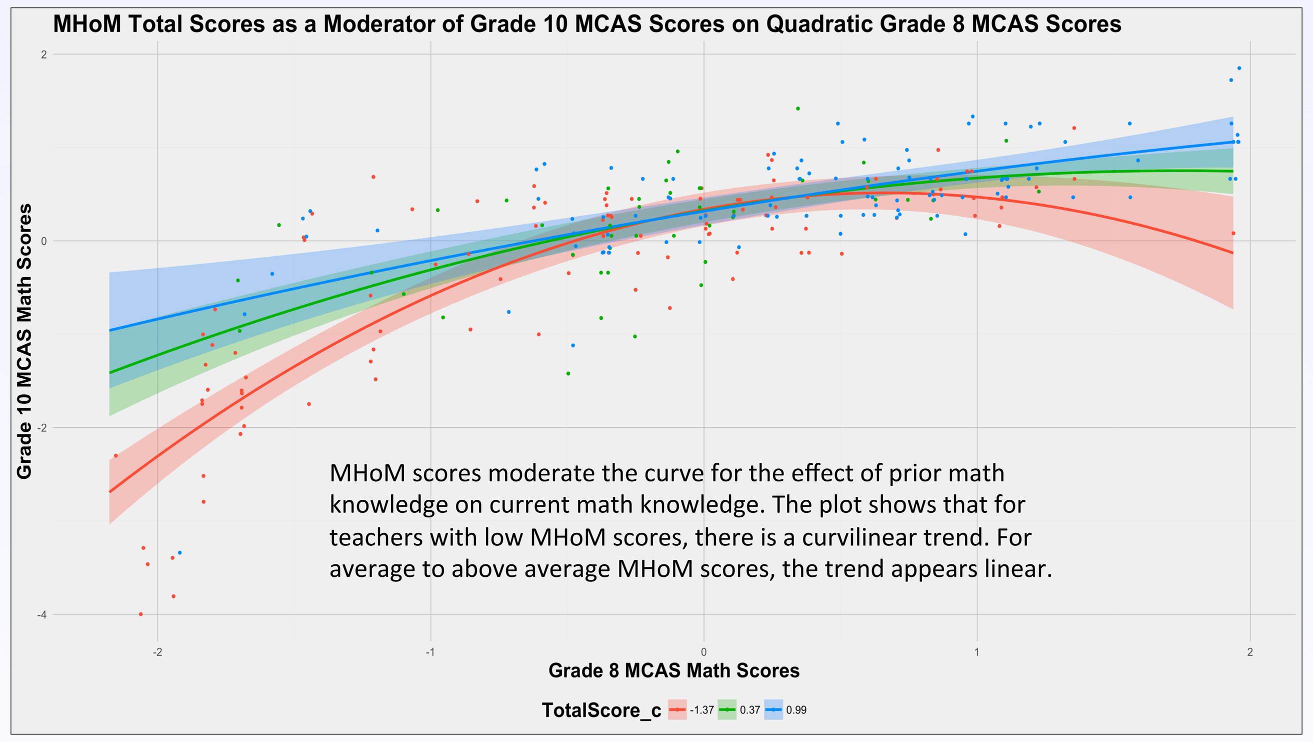
		C1	C2	C5	C6	C7
Change in Use of Structure	Pearson Correlation	.77**	.47	.56*	.56*	.59*
	N	13	13	13	13	13

C1: Caring; **C2:** Conferring; **C5:** Consolidating; **C6:** Challenging; **C7:** Classroom Management.

*Data collected and analyzed by the NSF-funded *Designing for Equity By Thinking in and About Mathematics* MSP. (Sword, Badertscher, Gates, in preparation).

Connections to Student Achievement Data

Relationships between teacher MHoM and student achievement are complex. However, analyzing data in a small sample suggests that stronger MHoM for teachers are correlated with better outcomes for students.



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